Wind Turbines for use in Cities and Suburbs: Basic Wind Physics

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Small wind turbines in the urban environment: Current Research at McMaster University

Nominal performance: Full-scale wind tunnel testing

Actual performance: Rooftop testing

Structural and vibrations: FEA, Measurements

CFD Analysis: Flow within turbine and around building

Kevin McLaren

Stephen Kooiman
Two types of wind turbines according to shaft orientation:

- **Horizontal axis wind turbine** (Most common, well known)
- **Vertical axis wind turbine (VAWT):**
  - Drag type ([Cup anemometers, Savonius rotors](#))
  - Lift type ([Darrieus turbines and Variations](#))
Power

- Basic physics - instantaneous power $P$
  $$P = c_p \frac{1}{2} \rho v_{wind}^3 A$$
  - $c_p$ power coefficient = turbine efficiency ranges $0 \rightarrow 0.59$
  - $\rho$ air density
  - $v_{wind}$ windspeed
  - $A$ turbine area
Power - add units

- Basic physics - instantaneous power $P$ (W)
  
  $$P = c_p \frac{1}{2} \rho v_{\text{wind}}^3 A$$

  - $c_p$ power coefficient = turbine efficiency ranges $0 \rightarrow 0.59$
  - $\rho$ air density (kg/m$^3$)
  - $v_{\text{wind}}$ windspeed (m/s)
  - $A$ turbine area (m$^2$)

eg 10m diameter horizontal turbine with a $c_p$ of 0.4 in 12 m/s wind

$$P = c_p \frac{1}{2} \rho v_{\text{wind}}^3 A = (0.4) \frac{1}{2} (1.2)(12)^3 \left( \frac{\pi 10^2}{4} \right) = 32570W = 32.6kW$$
Power Curve

- Plot of power as function of windspeed
- includes area $A$
- includes efficiency $c_p$
- note $P \propto v^3$
- cut-out / cut-in
- not entirely reliable

- RATED POWER
Turbine efficiency ($c_p$)

- Small wind efficiencies are lower than large turbines
  - $c_p \sim 0.2 - 0.35$
  - charlatan $c_p \sim 0 - 0.1$ or claim $>0.59$
  - is often function of windspeed

- Dirty air of cities (turbulence, unsteadiness) $c_p$ reduced
  - inherently
  - ability of controller to respond
Average power output

- Depends on turbine power curve
  - what turbine can produce in given windspeeds

- Also wind distribution
  - how windspeed varies throughout the year
  - Weibull distribution is a common fit to measured data

- note: due to \( v_{\text{wind}}^3 \), you cannot just use average windspeed
Average power output

- Multiply turbine power curve and windspeed distribution and add up for average power output.

- Multiply this by time under consideration for total power production (e.g., average power output \( \times 24 \text{ h/day} \times 365 \text{ days/year} \) = annual power supply in kW-h).
  - so average of 1 kW production gives 8760 kW-h in a year.
Wind Data

- Canadian Wind Atlas (www.windatlas.ca)
  - average windspeed, wind distribution at various heights
Urban wind

- Lower wind velocity due to larger ground roughness within urban environments
- Complex vortical flow structures over and around buildings
- High turbulence levels

Numerical Modelling of a Building with Normal Flow to the Building’s Face
Wind data (Urban)

- Urban installation is very location sensitive
  - hills, escarpments, trees, buildings

- Rough rules of thumb
  - clear of obstacles
  - higher is better
  - measure?

- Urban wind is unsteady and turbulent
  - affects (reduces) $c_p$ of turbines
Noise

- Noise is proportional to the fifth power of blade velocity
  - in horizontal turbines the fastest bit of the blades are the tips
  - practically all noise generated by tips
  - increase of 50% in tipspeed increases noise by a factor of \((1.5)^5\)
    ~ 8

- Variable between manufacturers
  - other factors: trailing edge sharpness, flutter
Vibration

- All structures (turbines, towers, buildings) have natural frequencies at which they want to vibrate
- Turbines vibrate
  - unbalanced rotation (1 per RPM)
  - uneven aerodynamic loads on blades eg tower pass, wind shear (3 per RPM)
- Most important that frequency of excitation does not co-incide with any natural frequency
Betz Limit

Mass flow \( \dot{m} = \rho c_2 A_2 \)

Power to disk \( P = \frac{W}{t} = \dot{m} c_2 \Delta c \)

Energy lost by wind \( P = \frac{1}{2} \dot{m} c_1^2 - \frac{1}{2} \dot{m} c_3^2 \quad \Rightarrow \quad c_2 = \frac{1}{2} (c_1 + c_3) \)

Plug in \( P = \frac{1}{2} \rho c_2 A_2 c_1^2 - \frac{1}{2} \rho c_2 A_2 (2c_2 - c_1)^2 \)

and define \( c_p \equiv \frac{P}{1/2 \rho A_2 c_1^3} = \frac{1/2 \rho A_2 c_2 \left[ c_1^2 - (2c_2 - c_1)^2 \right]}{1/2 \rho A_2 c_1^3} = \frac{4 \left[ c_2^3 - c_1 c_2^2 \right]}{c_1^3} \)

Find \( c_{p,\text{max}} \) by taking \( \frac{dc_p}{dc_2} = \frac{4}{c_1^3} \left[ 3c_2^2 - 2c_1 c_2 \right] = 0 \quad \Rightarrow \quad c_2 = \frac{2}{3} c_1 \)

\( \Rightarrow c_{p,\text{max}} = \frac{16}{27} = 0.593 \)
HAWT

Enercon Wind Parc at Calenzana in Corsica

Bergey Excel (10kW)
Drag type VAWT

- rugged
- (very) low efficiencies

Cup anemometer

Savonius stacked rotor

Source: AWEA website
Lift type VAWT

Sandia 34 m testbed
Darrieus “eggbeater”
Source: Sandia National Labs

QR5
Source: Quiet Revolution

Cleanfield 2.5 kW
Vertical Axis Wind Turbines

- Blades held by struts rotate about a vertical axis.
- Blades move into, across, with and across the wind direction on each rotation.
- Thrust depends on the *apparent* wind seen by the *rotating* blades.
Basic Aerodynamics of VAWTs

- True wind
- Rotational wind
- Total apparent wind
- Blade lift & drag
- Thrust
- True wind
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